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OMB No. 0704-0188

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1. A

3. REPORT TYPE AND DATES COVERED
FINAL 1 Sep 89 - 15 Jul 93

4. TITLE AND SUBTITLE
"SPATIO-TEMPORAL COMPLEXITY & LARGE SCALE STRUCTURES IN PROBLEMS OF CONTINUUM MECHANICS" (U)

5. FUNDING NUMBERS
61103D
3484/D7 (URI)

6. AUTHOR(S)
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8. PERFORMING ORGANIZATION REPORT NUMBER
AFOSR-TR- 94 0144

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NM
Bldg 410
Bolling AFB DC 20332-6448

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AFOSR-89-0507

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release;
Distribution unlimited

12b. DISTRIBUTION CODE
UL

13. ABSTRACT (*Maximum 200 words*)

We have investigated some difficulties in estimating dynamics from time-delay embeddings of experimental data that can be characterized as low-dimensional. A new procedure is developed to reduce noise by exploiting the properties of saddle periodic orbits on the reconstructed attractor. Most of these methods involve the estimation of a derivative form the data or in some way require a least squares estimate of the location of some portion of the attractor. Our work addresses some of the problems inherent in the estimation of dynamics from data, regardless of the type of model used to approximate the dynamics. These difficulties may arise from the fractal structure of the attractor and errors in all the observations. The problems persist regardless of the amount of available data and affect one's ability to determine an accurate local model of the dynamics, even when an accurate model should be obtainable in principle. Many of these problems can be circumvented by using as much dynamical information as possible in the formulation of the statistical relationship between the observations. Our attempt to do this involves the use of recurrent orbits to derive an accurate linear model of the dynamics in the vicinity of saddle periodic orbits on the attractor. We have applied our method to two experimental data sets from Taylor-Couette flows.

14. SUBJECT TERMS

15. NUMBER OF PAGES
14

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

AEOSR-TR- 94 0144

Final Report

AFOSR University Research Initiative on:

"Spatio-Temporal Complexity and

Large-Scale Structures in

Problems of Continuum Mechanics"

Department of Mathematics

Arizona State University

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Unannounced	☐	
Justification		
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ASU TKA 1758/TE

1. Infinite Dimensional Dynamical Systems: Exponential Attractors

The basic property of dissipative Partial Differential Equations (P.D.E.'s) is that their global asymptotic behavior is controlled by a finite number of parameters. A *global attractor* X exists, which is the largest compact set both positively and negatively invariant under the flow, and *uniformly* attracting all bounded sets in the phase space and is unique. One of the properties that makes the attractors an important object to study is the fact that often they have finite dimension.

Since the attractor is finite dimensional, it is natural to expect that it can be recovered by solving a large enough system of ODE's, that is, the solutions on the attractor satisfy a system of ODE's. An indirect way of obtaining such a system is to imbed the attractor into a finite dimensional smooth manifold. An *inertial manifold* is an exponentially attracting, finite dimensional, Lipschitz manifold that is invariant under the forward flow.

Fundamentally, we do not know the optimal rate of convergence for the trajectories of a dissipative PDE to its global attractor X. It need not be exponential; and we cannot yet construct a true inertial manifold for the Navier Stokes equations. We can still address the following problem: what is the smallest compact set which is forward invariant under the flow and which attracts at a *uniform exponential rate* all bounded trajectories? Can one define a generalized (i.e., with noncontinuous coefficients) system of ODE's on such a set? A. Eden, B. Nicolaenko and collaborators have (partially) resolved these questions through construction of *Inertial Sets* [10]; an Inertial Set (also called *Global Exponential Attractor*):

1) is compact and forward invariant under the flow; hence it contains the attractor X;

2) has a finite fractal dimension

3) it attracts at a uniform exponential rate all trajectories which start in a bounded initial ball.

We have constructed such sets for 2-D Navier Stokes equations (periodic boundary conditions), damped hyperbolic systems (Sine-Gordon, Klein-Gordon [11], Compressible Van der Waals gases with Korteweg capillarity phase change models [12]) and many other dissipative

equations. Like inertial manifolds, inertial sets are not unique.

Inertial sets possess a deeper and more practical property: they remain *more robust under perturbations and numerical approximations* than global attractors. We elaborate on this point, since the literature sometimes gives the wrong impression that attractors are robust under perturbations. One can only establish upper-semi-continuity of attractors for approximations of semigroups and partial differential equations. Specifically, if X_ϵ is the approximation to the attractor X, and the functional space is equipped with a norm $|\cdot|$:

$$\max_{x\in X_\epsilon}\left\{\min_{a\in X}|x-a|\right\} \leq \epsilon,$$

that is, there exists a spherical ϵ-neighborhood of X which contains the approximate X_ϵ. The reverse is not true. Similar problems plague the many constructions of approximate inertial manifolds: these are in the vicinity of the exact attractor only in the sense of upper-semicontinuity. Whereas, we prove, at least for classical Galerkin approximations, denoting by X and $\mathcal{M}$ the exact global attractor and inertial set; by X_ϵ and $\mathcal{M}_\epsilon$ the approximate ones:

$$\max_{u\in X}\left\{\min_{u_\epsilon\in \mathcal{M}_\epsilon}|u-u_\epsilon|\right\} \leq \epsilon$$

(that is, the exact attractor X is within an ϵ-spherical neighborhood of the approximate inertial set $\mathcal{M}_\epsilon$). Moreover:

$$\max_{u_\epsilon\in X_\epsilon}\left\{\min_{u\in \mathcal{M}}|u-u_\epsilon|\right\} \leq \epsilon$$

(that is, the approximate attractor X_ϵ is within an ϵ-spherical neighborhood of the exact inertial set $\mathcal{M}$). Essentially, we also prove that approximate and exact inertial sets are continuous with respect to the Hausdorff distance, modulo a time-shift (ϵ-dependent), at least for classical Galerkin approximations [10].

For inertially stable (in a sense to be detailed at the end of this section) numerical schemes, computed trajectories lie on approximate inertial sets. *What we effectively measure or compute are trajectories on inertial sets.* The latter contain the slow transients as well as

the global attractor. In the theory of dynamical systems, the slow transients correspond to slowly converging stable manifolds. Numerical simulations of infinite-dimensional dynamical systems often capture both the slow transients and parts of the attractor. After a large but finite time, the state of the system obtained from the numerical calculation may often be at a finite distance from the global attractor but at an infinitesimal distance to the inertial set. In this sense, we also call the inertial set an Exponential Attractor [10] to be consistent with the physical intuition. Specifically, after a short transient time the infinite-dimensional system is arbitrarily close to an Exponential Attractor whenever the latter exists.

The intrinsic interest of the exponential attractor for N-S turbulence lies in the fact that there is no natural single time scale for the N-S global attractor. In addition, the exponential attractor allows for a study of large intermittent deviations at small scales. This phenomenon of turbulent intermittency has been pre-excluded in the theory of inertial manifolds since, in the latter, small scales are assumed to be globally slaved to large scales, which implies no amplification of disturbances propagating towards small scales. Other time scales also appear in the analysis of the equations that leads to the existence of exponential attractors; these might or might not be related with the above mentioned scale. In this sense, the study of exponential attractors might shed light to those physically observed and thus relevant behaviors, no matter whether they are on the global attractor or not [10].

2. Coherent Structures in Physical Flows

Inertial sets are an appropriate tool for a deeper mathematical understanding of complex spatio-temporal structures in chaotic and turbulent flows. For Reynolds numbers (Re) substantially beyond the first transitions to instability, to what extent does the random occurrence of coherent large-scale events reflect intermittent dynamics on much lower dimensional manifolds? Generally, a most intriguing problem in the theory of hydrodynamic turbulence is the formation of large-scale structures in a flow performing random turbulent motion at small scales. Such coherent structures (C.S.) and small scale turbulence may be

viewed as the main features of a double structure: Although the C.S. are not especially energetic nor long lived, they are important in transport of heat, mass and momentum. Indeed, researchers realized that the C.S. (such as large eddies in turbulent shear flow) ought to have ***quasi-deterministic behavior.*** Lower dimensional dynamical systems mechanics can generate enhanced turbulent transport via the C.S.

We have extensively investigated 2D turbulent bursting flows which strikingly fit the phenomenological C.S. picture outlined above. These are generalized Kolmogorov flows with spatially periodic forcing [13]. Their (moderately) turbulent regimes are a paradigm for the provocative statement by M. Lesieur:

"... coherent structures emerge from chaos, under the action of an external constraint." Moreover, there is recent renewed interest in the actual experimental realization of these 2D flows, via electric or magnetic fields. The classical two-dimensional Kolmogorov flow is the solution of the 2D Navier-Stokes equation with a unidirectional force $\mathbf{f} = (\nu k_f^3 \sin k_f y, 0)$. It was introduced by Kolmogorov in the late fifties as an example on which to study transition to turbulence. For large enough viscosity, ν, the only stable flow is a plane parallel periodic shear flow $\mathbf{u}_0 = (\nu k_f \sin k_f y, 0)$, usually called the "basic Kolmogorov flow." The macroscopic Reynolds number of the basic flow is easily found to be $1/\nu$; this will be used later as a free parameter to define the bifurcation sequence. It was shown by Meshalkin and Sinai that large-scale instabilities are present for Reynolds numbers exceeding a critical value, $\sqrt{2}$. In a 2π-periodic box, the equations are:

$$\begin{gathered}\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p = \nu \nabla^2 \mathbf{u} + \mathbf{f}, \qquad \nabla \cdot \mathbf{u} = 0, \\ \mathbf{f} = (\nu k_f^3 \sin k_f y, 0), \qquad 0 \leq x, \quad y \leq 2\pi.\end{gathered} \tag{1}$$

The most interesting transitions occur at even higher Reynolds number; they lead to sparsely distributed bursts in time for a fairly large range of Reynolds number above a certain threshold, i.e., Re $\approx$ 20.8 for $k_f = 8$ [13]. The most striking feature of this transition is that the bursts generate substantial spatial disorder and drive developed turbulence. We

have also investigated such bursting regimes in *generalized Kolmogorov flows*, where the *force is an eigenfunction of the linearized Stoke's operator.* For instance, a forcing stream function of the type $\cos(k_f x)\cos(k_f y)$ generates a basic flow of square eddies. Generally, quiescent states associated to large scale coherent vortices dominate the dynamics between the bursts. The burst corresponds first to weak, then to strong interactions between these C.S., with turbulent dynamics at *much smaller spatial scales.* Cusped, near singular vortices are sharply localized both in space and in time, during the bursts. As the Reynolds number increases beyond 100, the *same dynamical regime persists*, with a striking role reversal: "bursting" regimes now become prevalent (with homogeneous shear turbulence at smaller scales) whereas the intermittencies now reflect brief reorganization of the flow around the larger-scale, more symmetric coherent vortices: we have "bursts of reordering"; remarkably, dynamics around the C.S. are now turbulent, not laminar (as for the lower Re). We have established that symmetry-breaking mechanisms are the prime engine behind such *persistent* (in Re) dynamics.

We establish both computationally and analytically that the Kolmogorov bursting regimes are linked with symmetry-breaking heteroclinic connections which generate persistent (in Re) homoclinic cycles [13]. The coherent vortices are invariant under isotropy subgroups of symmetries of the Kolmogorov flow. The heteroclinic connections correspond to invariant submanifolds with further reduced symmetries; they exchange the slow-stable and unstable manifolds of the coherent vortices.

For the Kolmogorov flows, small-scale dynamics prevail in a neighborhood of the heteroclinic connections, whereas large-scale dynamics are linked with slow-stable manifolds of the hyperbolic Tori (C.S.). This is another paradigm of transient dynamics on an inertial set. The ideal global attractor is made of the hyperbolic tori and their multiple heteroclinic connections. Theoretical dynamics should get closer and closer to the ideal geometric connections, and pseudo-periods between bursts should increase monotonically to

infinity. This is not observed in practice, where bursts occur randomly, and (numerical) noise throws the trajectories onto larger slow manifolds of the hyperbolic tori. So, for a very long time, (computationally, forever) dynamics fluctuate within a tubular neighborhood of the ideal geometric heteroclinic connections. These results might suggest slow transients on an inertial set. Even more, for larger Reynolds numbers, the heteroclinic connections are probably not strictly attracting (for $t \to +\infty$).

3. Proper Orthogonal Decomposition Methods

The Karhunen Loeve (K-L) analysis or Proper Orthogonal Decomposition has been used for quite some time now to extract coherent structures out of PDE simulations and experimental data [Lumley, Sirovich]. With support from this grant we have recently been able to show that this method is also a suitable tool to extract phase-space information out of large scale PDE simulations [1, 2]. In particular, we have analyzed large scale simulations of the Kuramoto-Shivasinsky equation and of 2-d Kolmogorov flow. We find that the method works especially well for an intermittent turbulent regime which shows laminar or regular behavior for some time which then is interrupted by spatio-temporally irregular burst phenomena. In this context, restricting the K-L analysis to the laminar data will determine the metastable structures (stationary or time dependent), whereas the analysis of the bursts may determine stable and unstable manifolds of these metastable states as well as linear spanning dimensions for the chaotic transient that is involved. We can also analyze periodic and quasi-periodic regimes deriving eigenfunctions that determine the dynamics and structure of these (quasi-)periodic cases. These are very important data for any kind of qualitative understanding of large scale spatio-temporally chaotic flows. It will facilitate the development of low dimensional ODE models that approximate the large scale dynamics of the flow. It helps to elucidate the role of symmetry and symmetric subspaces in these flows.

As part of his master thesis, our student, Randy Heiland, has developed a graphical front end for the necessary calculations on an SGI machine which makes maximal use of

its window and graphics capability and at the same time allows one to perform the very computationally intensive work on a more suitable configuration. The resulting software package that we call KL-TOOL can be called up from an ftp server. It was presented at Dynamics Days '92, the Oberwolfach conference on "Dynamics and Bifurcation," the Metz Days, and the Arizona Days in Los Alamos and attracted a lot of attention.

With the aid of this tool, we (D. Armbruster, P. Chossat, B. Nicolaenko) are currently developing a low dimensional model for Kolmogorov flow that allows us to identify the symmetries involved in the bursting regimes. In that way we hope to be able to rigorously prove that bursting in Kolmogorov flow is generated by structurally stable heteroclinic orbits.

One of the major topics at Dynamics Days '92 was the analysis and characterization of spatio-temporal complexity. This has so far not been successfully done for complex real data. A major project that D. Armbruster and E. Stone are currently pursuing involves the analysis of experimental data coming from the dynamics of flames. These data were supplied by Michael Gorman and show spatially and temporally changing cellular flames. A preliminary analysis on a small set of data was successful to identify a nontrivial temporally periodic flame state. Currently we are mimicking those data with our own synthetic data to test out ideas. We can show that the Karhunen Loeve analysis allows one to differentiate dynamics on different spatial scales. Furthermore, we can extract characteristic numbers for distinct spatio-temporal states which are closely related to Lyapunov exponents. This work will be presented at the pattern formation workshop at the Fields Institute. Further studies are underway which will use large quantities of data obtained by digitizing the videotape that shows the spatio-temporal evolution of the flames.

4. Targeting in Low Dimensional Dynamical Systems

Eric Kostelich has developed a procedure to rapidly steer successive iterates of an initial condition on a chaotic attractor to a small target region about any prespecified point on the attractor using only small controlling perturbations [3]. Such a procedure is called *targeting.*

Previous work on targeting for chaotic attractors has been in the context of one- and two-dimensional maps. Kostelich has shown that targeting can also be done in higher dimensional cases.

Ott et al. [4] introduced the idea that control of chaos could in some cases be attained by feedback stabilization of one of the infinite number of unstable periodic orbits that naturally occur in a chaotic attractor. Their method has been used to control a driven, flexible beam about a saddle fixed point in a laboratory experiment whose dynamical behavior was well approximated by a two dimensional map [5].

Romeiras et al. [6] recently extended these ideas and applied them to stabilize saddle periodic points in an attractor in four dimensions arising from a map that describes a kicked double rotor. They showed that control can be achieved (perhaps after several thousand iterations) by using only one control parameter, even when the attractor has two positive Lyapunov exponents (the Lyapunov dimension [7] of the attractor is 2.8).

Targeting is a slightly different version of the control problem for a chaotic system. We assume that we are given some initial condition on the attractor, and we wish to rapidly direct the resulting trajectory to a small region about some specified point on the chaotic attractor. Because of the inherent exponential sensitivity of chaotic time evolutions to perturbations, one expects that this can be accomplished using only small controlling adjustments of one or more available system parameters.

This was demonstrated theoretically and in numerical experiments for the case of a two dimensional map by Shinbrot et al. [8] and also in a laboratory experiment for which the dynamics were approximately describable by a one dimensional map [9]. The object of our paper is to present a new method of targeting and to demonstrate its applicability in systems of higher dimensionality than previously considered.

Grebogi, Ott and Yorke suggested the problem of how to apply a targeting type of control in a higher dimensional system. In his work, Kostelich considered the double rotor

map [3], which is four dimensional. Using the targeting procedure, typical points on the double rotor attractor can be steered to within 10^{-4} of a given target point on the attractor in an average of 35 iterations. Although there is more than one positive Lyapunov exponent, the control is achieved by making successive changes to a single parameter (here the strength of the kick).

Because the dimension of the double rotor attractor is about 2.8, the average distance between nearest neighbors in a subset of N points on the attractor scales as $N^{-1/2.8}$. This implies that about 10^{11} iterations of the map are required on the average to come within 10^{-4} of the target without the control. Since the control procedure described in [3] can steer the initial condition to within 10^{-4} of the target in about 10^2 steps, the method gets to the target about 10^9 times faster than the uncontrolled chaotic process. The method is demonstrated with a mechanical system described by a four dimensional mapping whose attractor has two positive Lyapunov exponents and a Lyapunov dimension of 2.8. The target is reached by making very small successive changes in a single control parameter. In one typical case, 35 iterates on average are required to reach a target region of diameter 10^{-4}, as compared to roughly 10^{11} iterates without the use of the targeting procedure.

The method works by setting up a Newton procedure to determine two successive small changes to the kick to steer a given iterate to the stable manifold of a point leading to the target. (A typical point x on the attractor has a *stable manifold* S associated with it. The set is stable in the sense that $||F^n(x) - F^n(y)|| \to 0$ as $n \to \infty$ whenever $y \in S$.) The stable manifold S is two dimensional for typical points x on the attractor because there are two negative Lyapunov exponents associated with x, and the plane spanned by the Lyapunov basis vectors associated with the negative exponents is tangent to S at x.

Although in principle it is possible to determine four successive changes to the kick to steer the point x to a chosen target point near $F^4(x)$, standard numerical methods do not work well because small distances between points approximately triple on each iteration of

the map (the largest Lyapunov exponent is about 1.2). Thus it is impossible to get a good linearization unless the orbit to be targeted is already extremely close to the given iterate. Instead, Kostelich developed a procedure to hit the stable manifold of a given attractor point that is only two iterates down from the starting point x.

We are now evaluating ways in which this kind of higher dimensional targeting procedure can be applied to other physical problems. For example, Kostelich's targeting procedure might be applied to an initial point near some chaotic attractor in order to steer it near a saddle fixed point. Then the methods of Grebogi, Ott and Yorke might be used to stabilize the trajectory around the saddle orbit. The saddle orbits of interest would be those that correspond to some desirable physical state; for instance, a laminar state in an otherwise chaotic system.

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